

Abelian monopoles in finite temperature lattice gauge fields: Classically perfect action, smoothing and various Abelian gauges

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Using the renormalization group motivated smoothing technique, the large scale structure of lattice configurations at finite temperature is characterized in terms of Abelian monopoles identified in the maximally Abelian, the Laplacian Abelian, and the Polyakov gauge. Abundance and anisotropy of monopoles at deconfinement and gauge invariant properties like local non-Abelian action and topological density are studied. Monopoles are predominantly found in regions of large action and topological charge, rather independent of the chosen gauge.

Confinement in non-Abelian gauge theories has a popular explanation within the dual superconductor picture. In this scenario the condensation of Abelian monopoles leads to confinement of color charges through a dual Meissner effect. The mechanism has been substantiated by a large number of lattice studies. Abelian monopoles in the confinement phase, obtained from Abelian projection in an appropriate gauge and representing links on the dual lattice, have been shown to percolate through the $4D$ volume and to be responsible for a dominant contribution to the string tension [1].

Studying creation operators of monopoles, evidence for their condensation was found, independently of the gauge chosen [2]. On the other hand, lengths and locations of monopole trajectories *do* depend on the selected gauge. In order to point out other potential differences between monopoles corresponding to various gauges, we concentrate here on aspects of temperature dependence at the phase transition and of gauge invariant characteristics such as action and topological charge. To be able to do this we have studied the semiclassical vacuum structure.

To resolve the semiclassical structure of gauge fields, mostly the ‘cooling’ method has been used. However, even improved versions of this

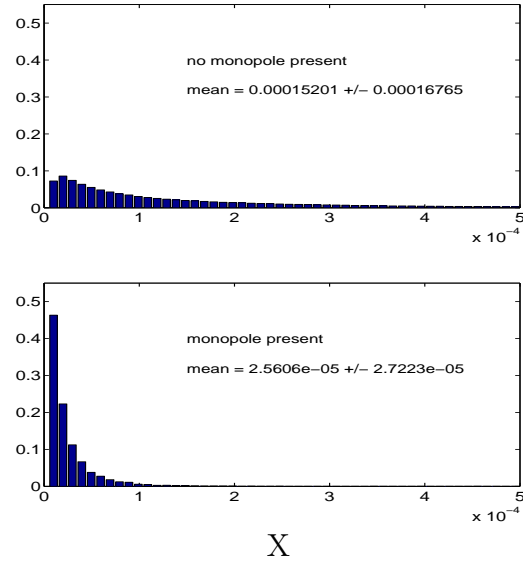


Figure 1. Probability distribution of the local norm $X = ||\Phi_x||$ of the auxiliary Higgs field of LAG, in the case of absence (top) or presence (bottom) of a DGT monopole.

method loose small instantons and instanton-antiinstanton pairs, and will destroy monopole percolation as well. To overcome these problems we employ a method of constrained smoothing [3] which is based on the concept of perfect actions [4]. This method does not drive configurations into classical fields but keeps large scale struc-

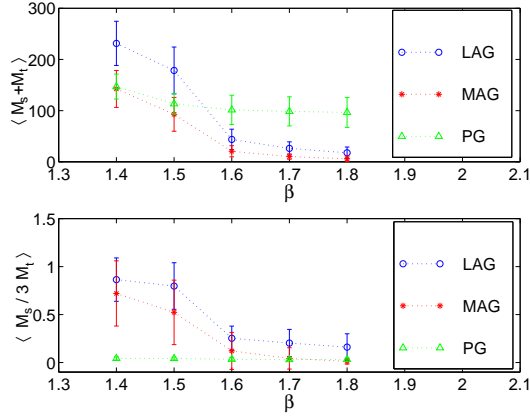


Figure 2. Total monopole length (top) and space-time asymmetry (bottom) as a function of β ($\beta_c = 1.545(10)$) for monopoles in different gauges.

tures as they are deformed by quantum interactions. There is a well-defined scale above which the semiclassical structure of the raw configuration is preserved¹.

We use a simplified fixed-point action [6] for Monte Carlo and for ‘constrained smoothing’ before a configuration is analyzed. Simulations were done on a $12^3 \times 4$ lattice. Observables were computed on 50 independent configurations, for each of the β values considered. The Abelian gauges considered here are the maximally Abelian (MAG) [7], the Laplacian Abelian (LAG) [8] and the Polyakov gauge (PG). The MAG can be considered as the minimization of

$$F[\Omega] = \sum_{x,\mu} \frac{1}{2} \text{Tr} (\Phi_x - U_{x,\mu} \Phi_{x+\hat{\mu}} U_{x,\mu}^\dagger)^2, \quad (1)$$

with the constraint $\|\Phi_x\| = 1$ where the gauge transformation Ω_x is encoded in $\Phi_x = \Omega_x^\dagger \sigma_3 \Omega_x$. The constraint on Φ is relaxed in the LAG, so that Eq. (1) can be interpreted as the kinetic term of an auxiliary adjoint Higgs field. Thus gauge fixing reduces to a lowest-eigenvalue problem for the covariant lattice Laplacian. If there is no degeneracy, LAG is globally unambiguous. As in MAG, fixing to LAG means diagonalizing

¹The iterative application of this method, ‘cycling’ [5], obscures the very idea of a definite blocking scale while it is still preserving long range physics rather well.

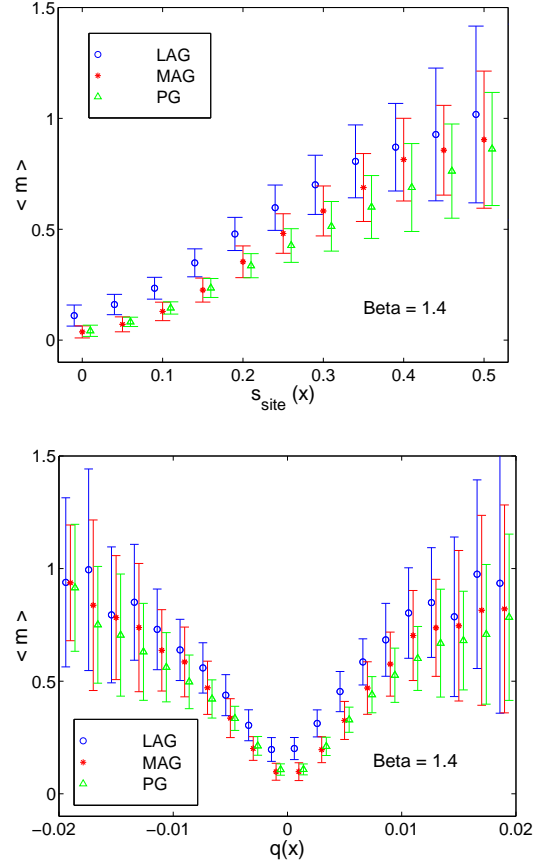


Figure 3. Average occupation number of monopoles $\langle m \rangle$ at sites with action density s_{site} (top) and topological charge density q (bottom) in the confinement phase.

Φ_x which is well-defined if $\|\Phi_x\| \neq 0$. (Similarly, PG is enforced by diagonalizing Polyakov lines.) DeGrand-Toussaint (DGT) monopoles are then defined by Abelian projection from LAG. We show in Fig. 1 that the zeros of the Higgs field corresponding to the gauge fixing singularities are correlated to the DGT monopoles where the Higgs modulus is strongly cut off.

Global properties like the total monopole loop length and the space-time asymmetry are compared in Fig. 2. Monopoles from MAG and LAG show a similar behaviour across the deconfinement phase transition. On the contrary, for the PG monopoles no change is seen in the neighborhood of the transition reflecting the fact that PG

monopoles are static in both phases.

In Fig. 3 we show the occupation probability of a monopole as a function of the local action $s_{site}(x)$ and charge $q(x)$ surrounding it, for the different gauges. This demonstrates that the probability of finding monopoles increases with the local value of action density/modulus of charge density. This result is practically independent of the gauge being used to construct the DGT monopoles.

We define an excess action of monopoles by

$$S_{ex} = \frac{\langle S_{monopole} - S_{nomonopole} \rangle}{\langle S_{nomonopole} \rangle}, \quad (2)$$

where $S_{monopole}$ is the action localized in a three-dimensional cube which corresponds to a dual link occupied by a monopole. Replacing the action by the modulus of topological charge according to Lüscher's definition we obtain the charge excess q_{ex} . For details of the definition of the local operators see [6]. Somewhat below T_c the excess action and charge for the MAG and LAG monopoles are above one, indicating an excess of action of more than a factor of 2 above background. The large error bars above T_c reflect the fact that the topological activity decreases in the deconfinement phase. Our results for the action excess in the confinement phase qualitatively agree with a recent study without smoothing [9].

Summarizing, we have provided evidence that Abelian monopoles are mainly localized in regions, which are characterized by enhanced action and topological charge. Therefore, monopoles can, at least to a certain degree, be viewed in a gauge invariant language as physical objects carrying action and topological charge.

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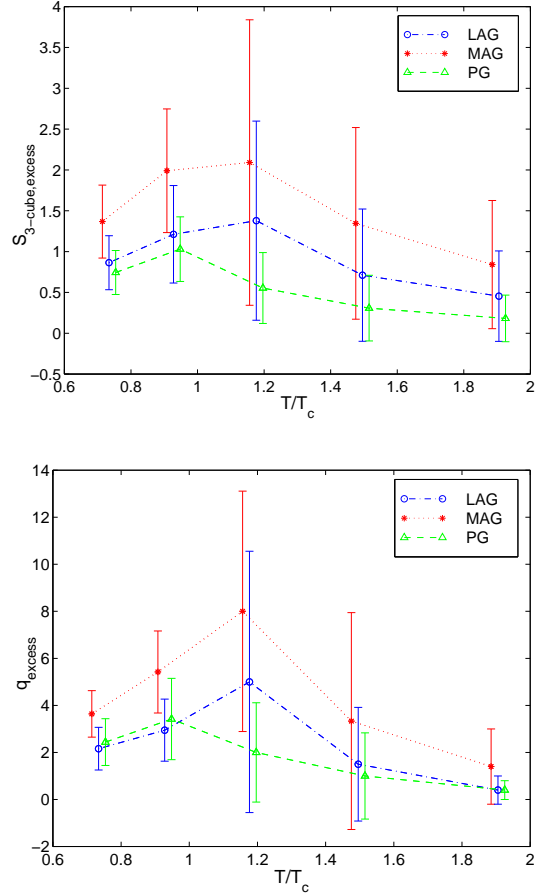


Figure 4. Excess action (top) and charge (bottom) as a function of temperature.

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